From the Excellent Book Werner Krauth- February 11, 2005

"Introduction To Monte Carlo Algorithms", we get some significant issues about how to use random numbers and games to evaluate complicated integrals and describe complex phenomena.
We read…
The word "Monte Carlo method" can be traced back to a game very popular in Monaco. It's not what you think, it's mostly a children's pass-time played on the beaches. On Wednesdays (when there is no school) and on weekends, they get together, pick up a big stick, draw a circle and a square as shown in figure. They fill their pockets with pebbles 1. Then they stand around, close their eyes, and throw the pebbles randomly in the direction of the square. Someone keeps track of the number of pebbles which hit the square,
and which fall within the circle (see figure). You will easily verify that the ratio of pebbles in the circle to the ones in the whole square should come out to be $\pi/4$, so there is much excitement when the 40th, 400th, 4000th is going to be cast.
This breath-taking game is the only method I know to compute the number π to arbitrary precision without using fancy measuring devices (meter, balance) or advanced mathematics (division, multiplication, trigonometry).
Children in Monaco can pass the whole afternoon at this game. You are invited 2 to write a little program to simulate the game. If you have never written a Monte Carlo program before, this will be your first one.

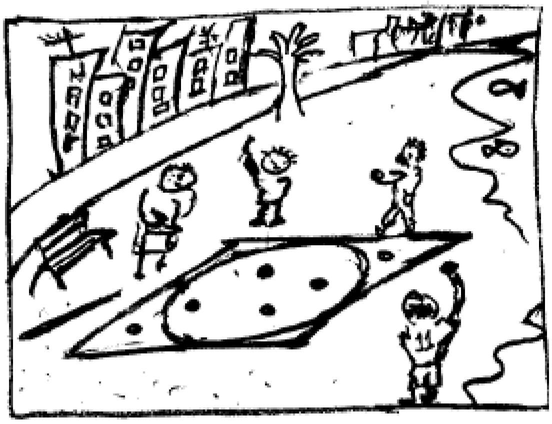

Figure. Children at play on the beaches of Monaco. They spend their afternoons calculating π by a method which can be easily extended to general integrals.

This method is fully explained in the book of Landau, R. H., Pαez, J., & Bordeianu, C. (2008). A Survey of computational physics introductory computational science. New Jersey: Princeton University Press.

Problem: Monte Carlo Integration by Stone Throwing
Imagine yourself as a farmer walking to your furthermost field to add algae-eating fish to a pond having an algae explosion. You get there only to read the instructions and discover that you need to know the area of the pond in order to determine the correct number of the fish to add. Your **problem** is to measure the area of this irregularly shaped pond with just the materials at hand .
It is hard to believe that Monte Carlo techniques can be used to evaluate integrals. After all, we do not want to gamble on the values! While it is true that other methods

are preferable for single and double integrals, Monte Carlo techniques are best when the dimensionality of integrations gets large! For our pond problem, we will use a *sampling* technique (see Figure)

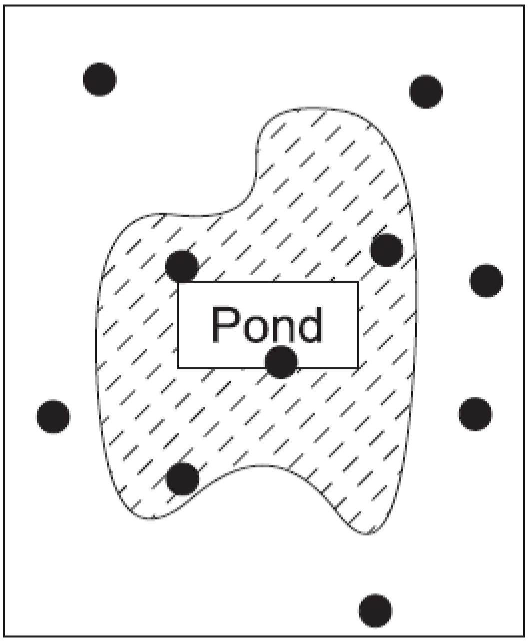

Figure: The sampling technique

The algorithm is described below:

1. Walk off a box that completely encloses the pond and remove any pebbles lying on the ground within the box.
2. Measure the lengths of the sides in natural units like *feet*. This tells you the area of the enclosing box.
3. Grab a bunch of pebbles, count their number, and then throw them up in the air in random directions.
4. Count the number of splashes in the pond and the number of pebbles lying on the ground within your box
5. Assuming that you threw the pebbles uniformly and randomly, the number of pebbles falling into the pond should be proportional to the area of the pond

You determine that area from the simple ratio

$$\frac{N_{\mathrm{pond}}}{N_{\mathrm{pond}} + N_{\mathrm{box}}} = \frac{A_{\mathrm{pond}}}{A_{\mathrm{box}}} \quad \Rightarrow \quad A_{\mathrm{pond}} = \frac{N_{\mathrm{pond}}}{N_{\mathrm{pond}} + N_{\mathrm{box}}} A_{\mathrm{box}}.$$

Figure(6.44 in the book of Landau et.al) The calculation of the area of the pond

To implement the algorithm we apply the following steps-they will be present at the Ejs Modeling:

1. Imagine a circular pond enclosed in a square of side 2 ($r = 1$).
2. We know the analytic area of a circle $\oint dA = \pi$.
3. Generate a sequence of random numbers $-1 \le r_i \le +1$.

4. For $i = 1$ to N, pick $(x_i, y_i) = (r_{2i-1}, r_{2i})$.
5. If $x_i^2 + y_i^2 < 1$, let $N_{\text{pond}} = N_{\text{pond}} + 1$; otherwise let $N_{\text{box}} = N_{\text{box}} + 1$.
6. Use (6.44) to calculate the area, and in this way π.
7. Increase N until you get π to three significant figures (we don't ask much that's only slide-rule accuracy).

Below we persent the algorithm in Ejs

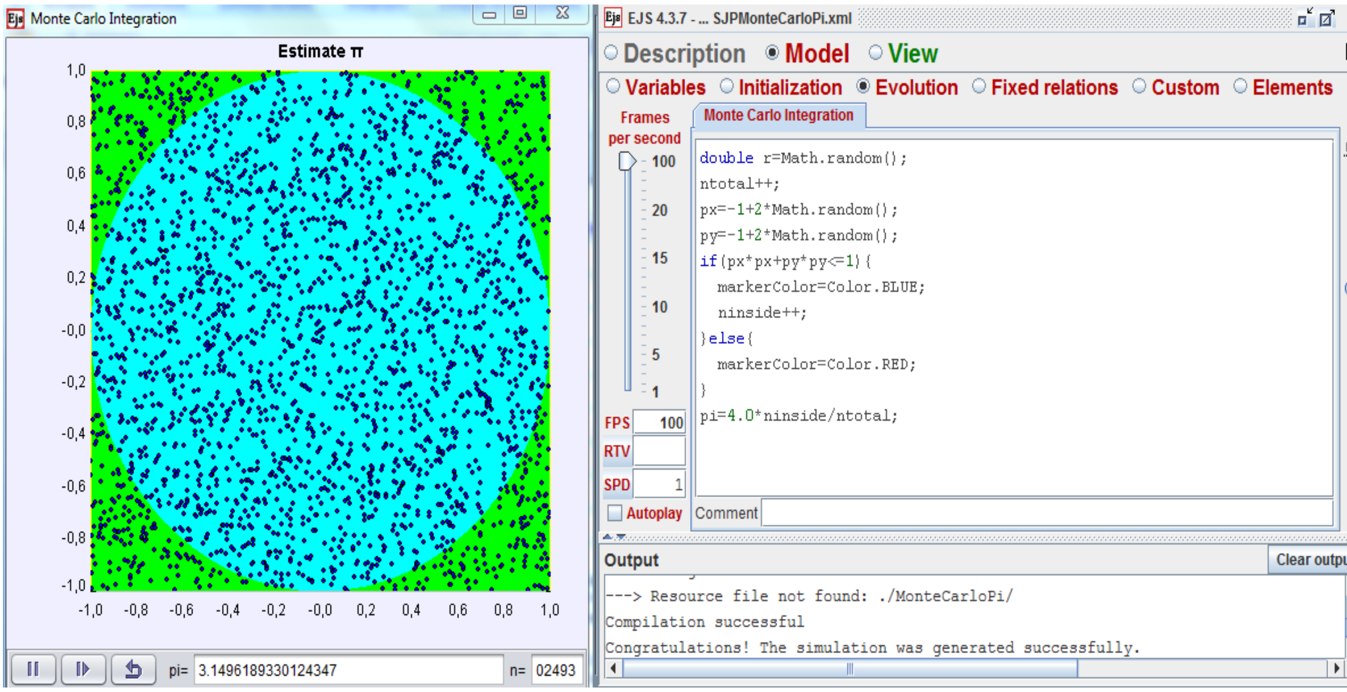

Figure. The algorithm in Ejs

In the algorithm above we generate a random variable r and the other two variables px,py and we examine if the sum of the squares of these variables lies inside the unit circle.